LAB 11

1. Prove the Unit, Zero, Idempotent, Law of Excluded Middle, and Commutative properties of ∥ stated above.
   1. Unit Property
      1. false || X = X
      2. it’s true if at least one of the operands is true
      3. since false is always false, the value depends only on X
   2. Zero Property
      1. true || X = true
      2. it’s true if at least one of the operands is true
      3. since true is always true, it doesn’t depend on X
   3. Idempotent Property
      1. X || X = X
      2. it’s true if at least one of the operands is true
      3. X || X is true if X is true, and false if X is false
      4. It depends only on X
   4. Law of Excluded Middle
      1. X || !X = true
      2. it’s true if at least one of the operands is true
      3. if X is true, then !X will be false so the value will be true
      4. if X is false, then !X will be true so the value will be true
   5. Commutative Property
      1. X || Y = Y || X
      2. the order does not matter, only the value of X and Y
2. Prove the two additional variations of De Morgan’s Law:
   1. X || Y = !(!X && !Y)
      1. !(!X && !Y) = !!X || !!Y = X || Y
   2. X && Y = !(!X || !Y)
      1. !(!X || !Y) = !!X && !!Y = X && Y
3. Prove (5), and (6) defined above.
   1. X || (Y && Z) = (X || Y) && (X || Z)
      1. If X is true:
         1. X || (Y && Z) is true
         2. (X || Y) is true
         3. (X || Z) is true
         4. (X || Y) && (X || Z) is true
      2. If X is false:
         1. X || (Y && Z) is true only if (Y && Z) is true, Y and Z both need to be true
         2. (X || Y) && (X || Z) becomes Y && Z, which is true if Y and Z are true
   2. X && (Y || Z) = (X && Y) || (X && Z)
      1. If X is true:
         1. X && (Y || Z) is true only if (Y || Z) is true, so either Y or Z need to be true
         2. (X && Y) || (X && Z) becomes Y || Z, which is true if either Y or Z is true
      2. If X is false:
         1. X && (Y || Z) is false
         2. (X && Y) is false
         3. (X && Z) is false
         4. (X && Y) || (X && Z) is false
4. Prove the Modus Ponens (8), Contrapositive (9), Shunting (10), and (a) and (b) below:
   1. X || (!X 🡺 Y) = X || Y
   2. X 🡺 Y && Z = (X 🡺 Y) && (X 🡺 Z)
      1. X && (X 🡺 Y) = X&& Y
         1. X 🡺 Y = !X || Y
         2. X && (X 🡺 Y) = X && (!X || Y) = (X && !X) || (X && Y)
         3. (X && !X) is always false so the value will depend only on (X && Y)
      2. X 🡺 Y = !Y 🡺 !X
         1. X 🡺 Y = !X || Y
         2. !Y 🡺 !X = Y || !X
         3. Due to commutativity it’s true
      3. X && Y 🡺 Z = X 🡺 !Y || Z
         1. X && Y 🡺 Z = !(X && Y) || Z = !X || !Y || Z = !X || ( !Y || !Z) = X 🡺 !Y || Z
      4. a)
         1. X || (!X 🡺 Y) = X || (X || Y)
         2. X || X is X
      5. b)
         1. X 🡺 Y && Z = !X || (Y && Z) = (!X || Y) && (!X || Z) = (X 🡺 Y) && (X 🡺 Z)
5. Prove the following formula: (P(x) && Q(y) 🡺 R(x, y)) && !R(x, y) && P(x) 🡺 !Q(y)
   1. P(x) && Q(y) 🡺 R = !(P(x) && Q(y)) || R(x,y)
   2. (!(P(x) && Q(y)) || R(x,y)) && !R(x,y)
   3. !R(x,y) is true
   4. !(P(x) && Q(y)) || R(x,y) && !R(x,y)
   5. R(x,y) && !R(x,y) is false
   6. !(P(x) && Q(y)) || false = !(P(x) && Q(y))
   7. !(P(x) && Q(y)) = !P(x) || !Q(y)
   8. P(x) is true
   9. !P(x) || !Q(y+ = !Q(y)
   10. (P(x) && Q(y) 🡺 R(x,y)) && !R(x,y) && P(x) 🡺 !Q(y)
6. Prove:
   1. (∃x : p(x)) && (∀x : p(x)) 🡺 ∃y : q(x, y)) 🡺 (∃x, y : q(x, y))
   2. (∃x : ∀y : P(x, y)) 🡺 (∀y : ∃x : P(x, y))